

Force Dynamics in Weakly Vibrated Granular Packings

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(Dated: February 2, 2008)

The oscillatory force F_b^{ac} on the bottom of a rigid, vertically vibrated, grain filled column, reveals rich granular dynamics, even when the peak acceleration of the vibrations is significantly less than the gravitational acceleration at the earth's surface. For loose packings or high frequencies, F_b^{ac} 's dynamics are dominated by grain motion. For moderate driving conditions in more compact samples, grain motion is virtually absent, but F_b^{ac} nevertheless exhibits strongly nonlinear and hysteretic behavior, evidencing a granular regime dominated by nontrivial force-network dynamics.

PACS numbers: 45.70.-n, 05.45.-a

Granular media consist of macroscopic solid grains which interact via dissipative, repulsive contact forces. Thermal energy is inconsequential, and granulates *jam* in random configurations unless sufficient mechanical energy is supplied, for example by shearing or shaking [1]. For sinusoidally, vertically vibrated granular media the driving strength is characterized by the nondimensional acceleration amplitude, $\Gamma = A(2\pi f)^2/g$, where A is the displacement amplitude, f is the oscillation frequency, and g is the gravitational acceleration at the earth's surface. In the well-studied case of $\Gamma \gtrsim 1$, grains periodically lose contact with and subsequently impact the oscillating container. The collision, as well as the accompanying relative motion between grains and wall, injects energy into the system that drives grain rearrangements [2, 3].

In this Letter we explore the $\Gamma < 1$ regime by measuring the oscillatory force on the container bottom, F_b^{ac} , in a weakly vibrated column filled with grains of total mass M [Fig. 1(a)]. F_b^{ac} exhibits rich dynamics. Under two conditions the dynamics is dominated by grain rearrangements: (i) Loose packings may compact when they are first subjected to weak vibrations, which leads to intermittent burst in F_b^{ac} . (ii) For large driving frequencies, grains slide periodically with respect to the container even at low Γ (e.g. $\Gamma \approx 0.1$ at $f = 900$ Hz), which leads to strongly nonlinear behavior in F_b^{ac} .

With moderate driving in compacted samples, however, *relative grain motion is minute but variations in the force configuration remain substantial* as indicated by nontrivial changes in F_b^{ac} . Weak vibrations thus excite strongly nonlinear and glassy dynamics of the force network. Such force variations are allowed, since for a terrestrial granulate the grain and deformation scales are separated by many orders of magnitude [4]: a 700 μm diameter bronze sphere is compressed only ~ 100 nm under the weight of 1000 additional identical spheres [5].

Figures 1(b,c) illustrate the main features of F_1 , the first harmonic of F_b^{ac} divided by Γ [see Eq. (1)], for a full column of grains undergoing slow triangular sweeps of Γ . For a solid mass placed on the bottom plate, F_1 is independent of Γ . For grains, however, F_1 depends strongly

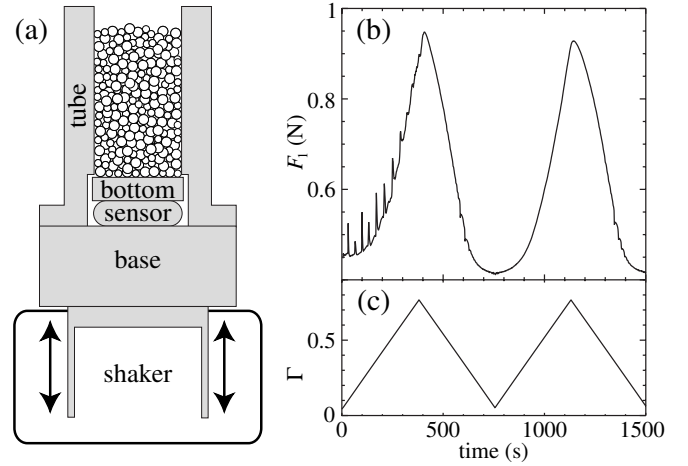


FIG. 1: (a) Schematic of the experiment showing the piezo force sensor mounted between the “bottom” and the “base” (diameter \times height: 32×12 mm and 89×62 mm respectively), with a cylindrical tube (inner/outer diameter 30/55 mm, height 113 mm) which is attached only to the base. The small ~ 100 μm gap between the tube and bottom plate prevents grains from becoming trapped. Shaded parts are rigidly connected and move in unison. The sensor signal is used to obtain F_b^{ac} , the vertical AC force exerted on the bottom plate by the grains. (b,c) Nonlinear response of a column filled with 200 g of 0.61 – 0.70 mm diameter bronze particles under sweeps of the vibration amplitude Γ for $f = 80$ Hz; F_1 denotes the calibrated ratio of the first harmonic of F_b^{ac} to Γ (Eq. 1).

and nonlinearly on Γ [e.g. $F_1(\Gamma = 0.5) \approx 2F_1(\Gamma = 0.05)$], even for low driving frequencies. The spikes in F_1 during the initial ramp are caused by compaction of the material. The asymmetry of $F_1(\Gamma)$ indicates hysteresis and memory in compacted samples. We stress that the strength of these features does not vary significantly for driving frequencies from 16 to 300 Hz. The phenomena evident in Fig. 1(b) are essentially *quasistatic* and are not associated with the excitation of sound waves (see below).

Experimental Setup — Nearly-spherical bronze particles sieved between 0.61 and 0.70 mm are poured into a smooth cylindrical tube with a detached bottom which is supported by a rigid piezo-electric force sensor (stiff-

ness 2.5 GN/m). The *entire assembly* is vertically oscillated with a small sinusoidal displacement [Fig. 1(a)]. An accelerometer attached to the tube measures the time-resolved acceleration $\gamma(t)$ which, for most driving conditions, is harmonic, equaling $\Gamma \sin(2\pi ft)$. The measured force is sensitive to temperature drift. The entire assembly is therefore placed in a temperature controlled enclosure maintained slightly above room temperature (temperature fluctuations ± 10 mK, humidity 5-10%); grains are equilibrated in the enclosure prior to use [6].

The deflection of the relatively compliant force sensors used in most previous studies is large compared to the deformation of hard grains like steel or glass; the granular force configuration is then completely altered due to relative motion between force probe and grains. In contrast, our tube/sensor assembly is effectively a *solid* container since the maximal deflection of the piezo is less than 1 nm, which ensures that the measured force variations are intrinsic to the granular medium [7, 8, 9].

In our experiment, the force sensor measures the total AC force, F_{total}^{ac} , which is the sum of the inertial force generated by the acceleration of the bottom plate and sensor, with effective mass M_0 , and the AC bottom force F_b^{ac} resulting from the acceleration of material in the column (which can have harmonics):

$$F_{total}^{ac} = \Gamma M_0 g \sin(2\pi ft) + \Gamma \sum_{n=1}^{\infty} F_n \sin(2n\pi ft - \phi_n). \quad (1)$$

To calibrate the signal, F_{total}^{ac} was measured for a range of f and Γ , both with and without solid test masses attached to the bottom plate. The value of F_{total}^{ac} for the empty system allows us to subtract the term $\propto \Gamma M_0 g$, after which F_1 is found to be proportional to the test mass and independent of Γ (the definition of F_n isolates the trivial scaling with Γ). The higher harmonics F_2, F_3, \dots are negligible in this case.

We now consider the AC vertical components of the frictional wall-force F_w and the bottom force F_b in a grain filled column. We will distinguish a *contact regime* where grains do not slide with respect to the column, and an *impact regime* where they do. In the impact regime,

$$Mg\Gamma \sin(2\pi ft) = F_w^{ac} + F_b^{ac}, \quad (2)$$

and our measurements probe how the vibration induced variations of the grain weight, $Mg\Gamma \sin(2\pi ft)$, are distributed between wall and bottom. However, Eq. (2) is violated in the impact regime. We have checked the validity of Eq. (2) by placing a separate, *closed bottom*, grain filled container directly on the bottom plate [10], so that the sensor measures the sum $F_w^{ac} + F_b^{ac}$; when the sensor signal remains purely harmonic, the system is in the contact regime, when (strong) nonlinearities are detected, it is in the impact regime. As we will show for the contact regime, F_b^{ac} grows nonlinearly with grain mass and Γ ,

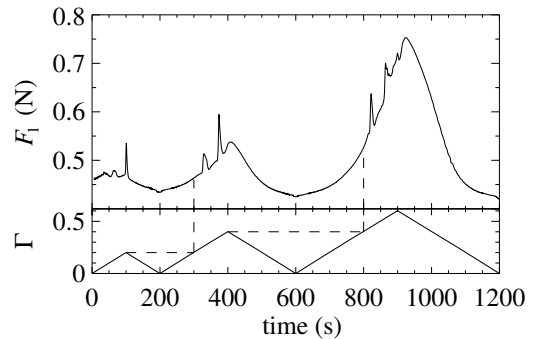


FIG. 2: Spikes observed for $f = 80$ Hz and $M = 200$ g in an initially low density packing. Sweeps of increasing magnitude in Γ illustrate that spikes only occur in “fresh” territory.

exhibits hysteresis and memory effects, and higher harmonics play a role—very different behavior than for a solid mass. However, we first discuss grain motion dominated *compaction* and *impact*.

Compaction — “Spikes”, such as those shown in Fig. 1(b), occur when Γ is ramped up in loosely packed samples, which are formed by placing the end of a funnel on the bottom of the container, filling the funnel with material, and then slowly retracting the funnel. Figure 2 illustrates that spikes only occur in “fresh” territory, *i.e.*, when Γ is increased beyond its previous maximum value. After Γ has been swept up to a value near one, we refer to the sample as *fully annealed*.

During a spike, which typically lasts for 1000’s of oscillation cycles, ϕ_1 shifts significantly (indicating dissipation) and F_b^{ac} is strongly non-sinusoidal. The specific Γ values where spikes occur vary from run to run, and hence are not resonant effects. Spikes are apparently due to compaction, since the free surface is lower after a spike occurs, and gently poured columns with lower initial density produce more spikes than less gently prepared ones with higher initial density. Spikes and their dependence on Γ apparently result from the full mobilization of frictional forces at the walls in granular columns [7, 11]. Under these conditions, vertical frictional forces are near their maximal values so that vibrations can cause (micro) slippage, and stronger vibrations may cause further compaction. The behavior of spikes is not frequency dependent: qualitatively similar spikes occur for f between 16 and 300 Hz, and switching to a different frequency in a partially annealed sample does not appreciably alter the Γ value separating fresh and annealed states [6].

Impact vs Contact — For compacted samples we distinguish between the contact regime where Eq. 2 is satisfied, and the impact regime where Eq. 2 is violated. Figure 3(a) illustrates that for low frequencies F_1 increases smoothly with Γ , while for higher frequencies there is a sudden upturn and a peak. In the vicinity of and above this transition, $\gamma(t)$ and F_b^{ac} are strongly anharmonic — as when grains periodically *collide* with the bottom for

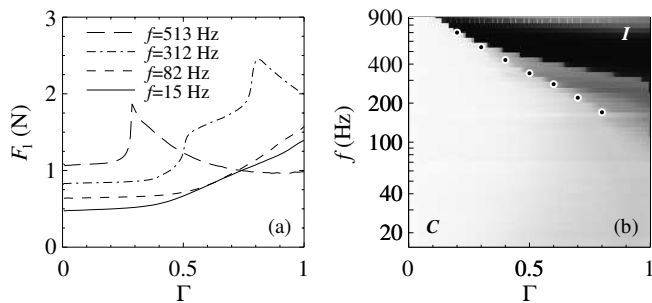


FIG. 3: Contact and impact regimes ($M = 200$ g): (a) $F_1(\Gamma, f)$ is smooth at low frequencies, but increases abruptly and peaks for higher frequencies (data is vertically offset for clarity). (b) Greyscale intensity plot of nonlinearity $\mathcal{N}(\Gamma, f) = F_1(\Gamma, f) - F_1(0, f)$ (white, $\mathcal{N}=0$; black, \mathcal{N} positive) indicating the contact (C) and impact (I) regimes. Dots indicate the initial sharp increase in F_1 .

$\Gamma > 1$; this is the impact regime. Below the transition, $\gamma(t)$ remains sinusoidal; this is the contact regime.

To check that Eq. (2) is valid in the contact regime (*i.e.* no slipping) but not in the impact regime, we replaced the open-bottom tube with a similarly sized, but *closed-bottom* grain filled tube mounted directly on the bottom plate. In this configuration, F_b^{ac} is insensitive to smooth force transfers between the wall and the container bottom which allows Eq. (2) to be checked directly. In the impact regime F_b^{ac} and $\gamma(t)$ are strongly anharmonic in violation of Eq. (2); in the contact regime they remain sinusoidal with F_1 essentially independent of Γ in agreement with Eq. (2). The physics underlying the force dynamics in the contact regime is thus a smooth, periodic transfer of grain weight between wall and bottom.

Figure 3(b) displays the strength of the nonlinearity of F_1 as function of Γ and f —the rapid increase in nonlinearity marks the onset of impact. Note that at high frequency ($f \approx 1$ kHz), the impact regime occurs for surprisingly small Γ (≈ 0.1). Apparently, impact for $\Gamma < 1$ is due to the excitation of resonant granular sound waves. Typical sound speeds are of the order of 100 ms^{-1} [12], so in our 10 cm deep column we expect a resonant response around 1 kHz. This picture is consistent with findings of Yanagida *et al.* [10] in studies of the resonant response of grain filled closed bottom containers for small Γ , and it is also consistent with the shift of the impact transition to higher frequencies for smaller M (this also excludes a trivial resonance of the apparatus[6]). The Γ dependence of this transition is not fully understood.

Contact Regime Nonlinearity — We now explore the nonlinear response of fully annealed samples in the contact regime as a function of M and Γ . The frequency is fixed at 80 Hz, since in the contact regime the grain response varies only weakly with f .

Figure 4(a) illustrates that the small mass behavior is independent of Γ , and that the grains are supported entirely by the bottom since $F_1 \approx Mg$. For larger masses,

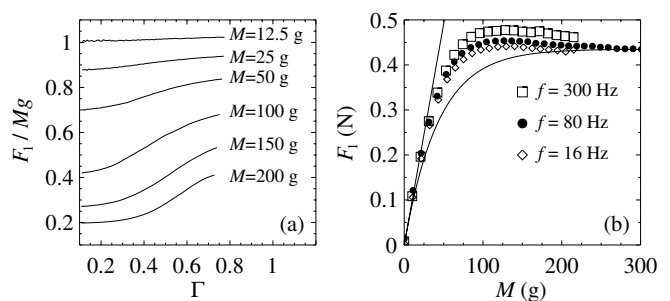


FIG. 4: Nonlinearity of F_1 with Γ and total grain mass M . (a) $F_1(\Gamma)/Mg$ for various filling fractions ($f = 80$ Hz). For larger M and fixed Γ , the weight fraction on the bottom decreases. (b) F_1 as function of M in the linear regime ($\Gamma = 0.05$) for $f = 16, 80$ and 300 Hz, compared to a linear response (straight line) and a Janssen-like response $F_1/F_{sat} = 1 - \exp(-Mg/F_{sat})$ for $F_{sat} = 0.435$ N.

wall forces start to play a role since $F_1 < Mg$ and the material's response becomes increasingly nonlinear with Γ : the exchange between bottom and wall forces underlies the nonlinearity of $F_1(\Gamma)$ for large M . This strong nonlinearity is accompanied by a modest increase in the harmonic distortion [13] of F_b^{ac} and a similarly modest shift in ϕ_1 . For example for $\Gamma = 0.7$ and $M = 200$ g, F_1 is doubled from its low Γ value, while the harmonic distortion is only 6% and $\phi_1 \approx 6^\circ$. Underlying the harmonic distortion is the nonlinear (Hertzian) contact-force law which makes grain contacts stiffer when under larger pressure [5]; the waveform distortions we have obtained for $F_b^{ac}(t)$ are consistent with this picture [6].

For small Γ the response is linear in Γ : for $\Gamma \approx 0.05$, F_b^{ac} is harmonic ($< 1\%$ distortion) and in-phase with the acceleration, and F_1 varies less than 1% for $0 < \Gamma < 0.1$. We study $F_1(M)$ as a function of increasing mass M at $\Gamma \approx 0.05$ by incrementally pouring grains from a height of approximately 10 cm above the grain surface. Figure 4(b) illustrates that F_1 grows proportionally with M for small masses, but then rapidly saturates to $F_1^{sat} \simeq 0.435$ N. $F_1(M)$ is only weakly frequency dependent, again indicating that in the contact regime a well-defined quasi-static regime is probed. Note that for all f a small overshoot occurs for intermediate values of $M \approx 100$ g.

This behavior is reminiscent of the Janssen effect for which the DC bottom force F_b^{dc} goes as [7, 11, 14]:

$$F_b^{dc} = F_{sat}^{dc} [1 - \exp(-Mg/F_{sat}^{dc})], \quad (3)$$

where F_{sat}^{dc} is the saturation force. Figure 4(b) shows, however, that the AC bottom force F_1 significantly deviates from the static Janssen result. We conclude that, even in the limit of weak vibrations, the force variation F_b^{ac} is *not* simply related to the steady force F_b^{dc} .

Hysteresis and Memory — So far, our data is consistent with a simple nonlinear response of the material, but Fig. 5(a) indicates that when Γ is ramped up and down in the nonlinear regime, F_b^{ac} is also hysteretic. The

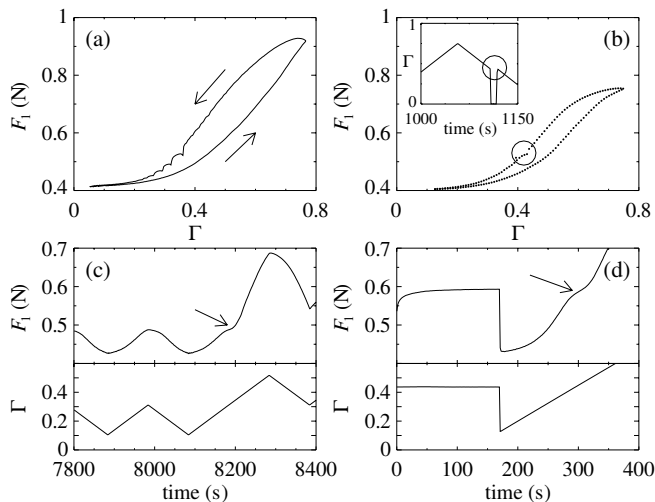


FIG. 5: Memory effects ($M = 200$ g and $f = 80$ Hz). (a) Hysteresis loop. (b) Hysteresis persists when Γ is suddenly set to zero and then rapidly ramped back up, indicated by the circle (see inset for details of ramp). (c-f) Two examples of subtle memory effects.

magnitude of hysteresis is only weakly dependent on the driving frequency and increases with M and Γ similar to the magnitude of overall nonlinearity. The hysteresis is nearly independent of the sweep speed (for sweep durations longer than $\approx 100/f$), and the force configuration thus depends on the driving history. Fig. 5(b) illustrates that these configurations can be “frozen”, since the system returns to the upper branch of the hysteresis loop after the driving is switched off and then rapidly ramped up again. The system exhibits additional subtle memory effects: after a fully annealed system is subject to a number of small amplitude sweeps in Γ , and then Γ is ramped beyond the peak value of the small sweeps, a clear “kink” in the F_1 curve is exhibited [Fig. 5(c,d)]. When a fully annealed system is driven at a fixed Γ , it “remembers” this value when Γ is rapidly decreased and then ramped past the initial fixed value [Fig. 5(e,f)].

Discussion — Our experiments exhibit rich dynamical behavior of weakly excited ($\Gamma < 1$) granular media which can be dominated by either grain motion or by contact force variations. That grain motion and compaction occur in loose samples vibrated at low frequencies and $\Gamma < 1$ is maybe not surprising, although we are unaware of systematic studies of compaction in this regime [3]. In compacted samples, both grain dominated (impact) and force dominated (contact) regimes can be distinguished with f playing a crucial role. For low f , the system is in the contact regime, and we observe little systematic variations with f . We therefore interpret the nonlinearities, memory effects, and hysteresis of the forces in response to variations in the driving strength as quasistatic, and not related to the excitation of sound waves. We have not identified any theoretical or numerical descriptions

of these surprisingly strong effects. Exploratory experiments in various columns with rough walls and for particles of different sizes further illustrate the robustness of these phenomena [6].

How should weakly vibrated granular systems be viewed? The contact laws between elastic bodies display a variety of nonlinear and hysteretic behaviors [5, 15]. A weakly driven granular assembly apparently amplifies the local nonlinearities present in the hertzian contacts and the friction law. We propose that force networks “activated” by weak vibrations explore many different configurations consistent with the overall boundary conditions for the stress [4, 16]. Such activated force networks could possibly play a role in creep flows, which occur far away from shear zones, and more generally in any granular system in which tiny relative grain motions are excited. In this sense, weakly driven granulates cannot be thought of as ordinary solids.

The authors thank CATS and the Netherlands Organization for Scientific Research (NWO) who supported numerous visits to NWU during which this work was carried out.

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